SENSITIVITY OF THE OPTIMUM BUCKET TRAJECTORY IN CONTROLLED EXCAVATION

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1. INTRODUCTION.

In recent years there has been increasing interest in the development and deployment of excavation automation. Several works has been devoted to the controlled motion of an excavator bucket along assumed trajectory. Among them are works by Bernold [1], Bradley and Seward [2], Hiller and Schneider [6,8]. However, some difficulties in realization of such a motion with prescribed accuracy has been observed. This can be caused, among others, by discontinuous motion of hydraulic actuators. The latter are moving according to the openings and closings of hydraulic valves distributing the oil. Electro-modulated hydraulic systems applied in modern excavators embraces both, electro-hydraulic controls, and servo-systems operating as a close-loop with feedback. In any open-loop electro-modulated system input is provided by a reference voltage fed to an electrical regulator, passing on information to an electro-servo valve. The latter in turn causes the motion of hydraulic actuator. Good response and correlation is usually assured for systems with constant parameters. This is because the electro-hydraulic systems cannot react properly to any discrepancies between input signal and resulting output. In the cases when constant values cannot be assured, then differences between input signal and the assumed actuator motion can occur. The discrepancy can be minimized in close-loop electro-modulated systems. This is due a transducer added, allowing the output to provide a feedback signal directed to the input side. However, also in this case mechanical inertia of the system may give not the expected output. A significant discrepancy between inputs and resulting outputs can be also caused by unpredicted, sharp variations of soil properties, such as stones or other obstacles. A rapid variation of a force on the bucket tip may cause change of the oil pressure in actuators and then change the assumed output of hydraulic system. In this paper, the sensitivities of the bucket motion with respect to small variation of hydraulic actuators' lengths are discussed. The problem is investigated for kinematically induced digging process performed by a backhoe excavator. It is assumed that all three actuators of the machine can work independently and simultaneously. This gives possibility of moving the bucket, along three degrees of freedom independently. Other words, controlling the flow of hydraulic oil into actuators it is possible to control motion of the bucket in a unique way. The discussed processes are of dynamic nature, (Vähä and Skibniewski [9,10]) and sensitivity analysis should incorporate dynamic analysis of the system. However, not all phenomena occurring here are known, and their mathematical model can not be incorporated into our discussion. This is the reason for which only kinematic analysis of the sensitivities is considered. The sensitivity analysis is based on the variation of actuator lengths. The latter, of course, depends on dynamic behaviour of the system, which means that presented results are having evaluative significance.

The first section of the paper is devoted to the derivation of implicit functions joining components of the bucket motion with actuator lengths. The former is two displacements of the bucket edge, and its rigid rotation. The second section deals with discussion of small changes of the bucket coordinates, in terms of variations of actuator lengths. Applying the rule of chain differentiation, we arrive to the discussed sensitivities. In the next section, evaluation of small actuators' motions is discussed, in view of openings and closing of proportional valves. The latter are opened and closed in finite distance of time, depending upon the finite lengths to which the prescribed trajectory is divided. The last part of the paper is presenting numerical examples of sensitivities of discussed coordinates. Calculations deal with sensitivities of coordinates defining an optimum trajectory.

2. MOTION OF THE BUCKET AND ACTUATOR LENGTHS.

A backhoe excavator (Fig 1) with boom, arm and bucket of lengths l1, l_2, and l_3 respectively is considered. Three hydraulic actuators of length h1, h2, and h3 drive the linkage. The angle between boom and horizontal line is • $_1$.The relative angles between boom and arm, as well as between arm and bucket are • $_2$, and • $_3$ respectively. Assuming motion of the bucket in x – z plane, the three degrees of freedom of the bucket (two displacements and a rotation) can be expressed as follows:

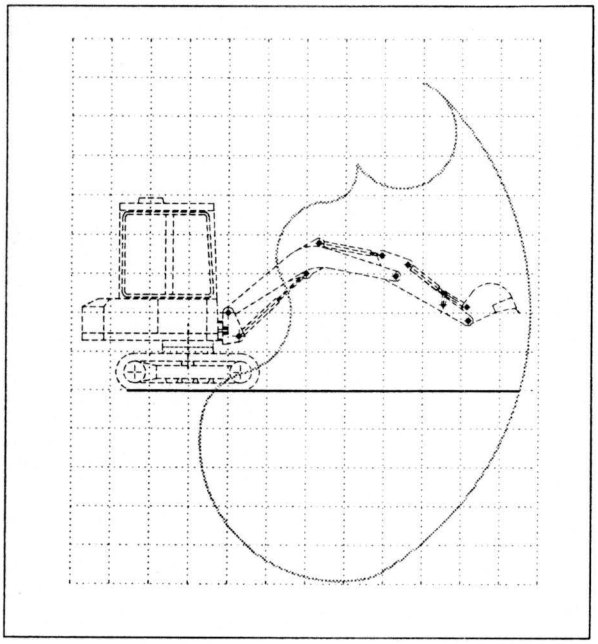

Fig. 1. A backhoe excavator considered

$$x = l_1 \cdot \cos(\alpha_1) + l_2 \cos(\alpha_1 + \alpha_2) + l_3 \cos(\alpha_1 + \alpha_2 + \alpha_3) \quad (1)$$

$$z = l_1 \cdot sin(\alpha_1) + l_2 sin(\alpha_1 + \alpha_2) + l_3 sin(\alpha_1 + \alpha_2 + \alpha_3) \quad (2)$$

$$\frac{\partial z}{\partial x} = -c\,\mathrm{tg}(\alpha_1 + \alpha_2 + \alpha_3 + \delta) \quad (3)$$

It is assumed that the angle of rotation of the bucket is oriented with respect to the tip trajectory according to the relation (3). Equations (1)-(3) are relating the position of the bucket with respect to three angles • $_1$, • $_2$, and • $_3$. Further considerations of the discussed sensitivities require explicit relations between the actuator lengths and angles • $_1$, • $_2$, and • $_3$. Inspecting Fig. 2, 3, 4 and taking from their dimensions defining positions of actuator attachments can do this. This way we find the length h_1 to be equal to:

$$h_1^2 = a_0^2 + a_1^2 + b_0^2 + 2(a_1 b_0 - a_0 b_1)\cos\alpha_1 - 2(a_0 a_1 + b_0 b_1) sin\alpha_1 \quad (4)$$

The length h_2 equal to:

$$h_2^2 = a_2^2 + b_2^2 + c_2^2 + 2b_2\left[a_2 sin(\gamma_0 - \alpha_2) + c_2 \cos(\gamma_0 - \alpha_2)\right] \quad (5)$$

The relation expressing h_3 is more complex. In this case it is convenient to introduce an additional unknown • and to give the length of the third actuator by means of two equations (6) and (7):

$$h_3^2 = a_3^2 + c_2^3 + d^2 + g^2 - 2a_3 g - 2d(a_3 - g) sin\beta + 2cd\cos\beta \quad (6)$$

$$b_3^2 = d^2 + e^2 + f^2 + g^2 + 2d\left[g - esin(\mu - \alpha_3)\right]sin\,\beta - 2egsin(\mu - \alpha_3) + - 2d\left[f + e\cos(\mu - \alpha_3)\right]\cos\beta + 2ef\cos(\mu - \alpha_3) \quad (7)$$

The system of equations from (1) to (7) is relating all geometric values needed to find sensitivities in question. The displacement z is considered to be a function of all other variables entering in these equations.

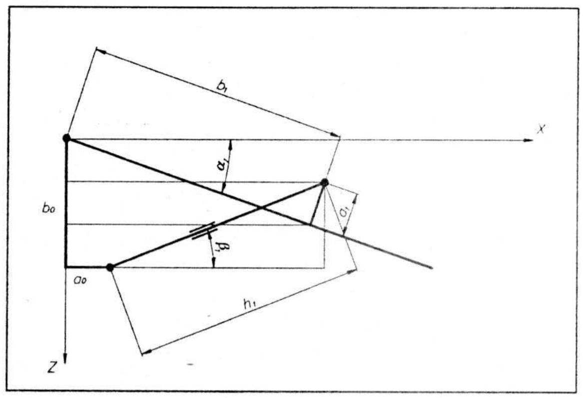

Fig. 2. Attachments of the actuator h1.

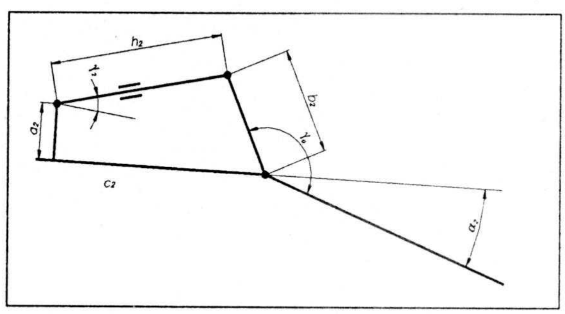

Fig. 3 Attachments of the actuator h2.

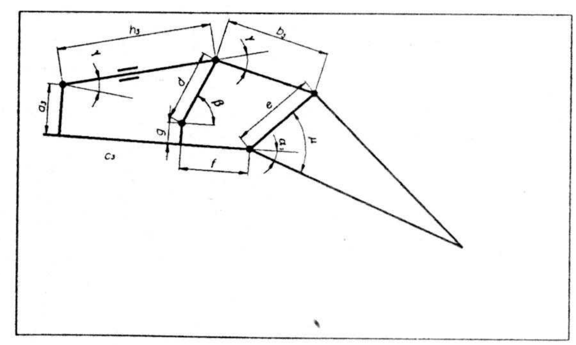

Fig. 4. Attachments of the actuator h2.

3. SENSITIVITY OF THE BUCKET POSITION WITH RESPECT TO ACTUATOR LENGTHS

The sensitivity of a function with respect to a variable can be considered as a total derivative of the function with respect to this variable. In discussed case the sensitivity of z with an arbitrary h_i $(i = 1,2,3)$ can be presented as follows:

$$\frac{dz}{dh}=\frac{\partial z}{\partial h}+\frac{\partial z}{\partial x}\frac{\partial x}{\partial h}+\sum_{j=1}^{3}\frac{\partial z}{\partial \alpha_j}\frac{d\alpha_j}{dh_i}+\frac{\partial z}{\partial \beta}\frac{d\beta}{dh_i} \quad (8)$$

This expression is containing, moreover, the total derivatives of variables x, • $_i$ $(i=1,2,3)$ and • with respect to h_i. They can be described from equation (4) – (7) as follows:

$$\frac{d\alpha_1}{dh_1}=-\frac{h_1}{(a_1b_0-a_0b_1)sin\alpha_1+(a_0a_1+b_0b_1)\cos\alpha_1} \quad (9)$$

$$\frac{d\alpha_2}{dh_2}=-\frac{h_2}{b_2\left[c_2 sin(\gamma_0-\alpha_2)-a_2\cos(\gamma_0-\alpha_2)\right]} \quad (10)$$

$$\frac{d\alpha_3}{dh_3}=\frac{d\left[g+f+esin(\mu-\alpha_3+\beta)\right]}{desin(\mu-\alpha_3-\beta)+e\left[g\cos(\mu-\alpha_3)+fsin(\mu-\alpha_3)\right]} \quad (11)$$

$$\frac{d\beta}{dh_3}=-\frac{h_3}{d\left[(a_3-g)\cos\beta+csin\beta\right]} \quad (12)$$

$$\frac{d\alpha_3}{dh_3}=\frac{\partial\alpha_3}{\partial h_3}+\frac{\partial\alpha_3}{\partial\beta}\frac{d\beta}{dh_3} \quad (13)$$

$$\frac{\partial z}{\partial h_i}=\frac{\partial x}{\partial h_i}=\frac{\partial z}{\partial\beta}=0 \quad (14)$$

Substituting equation (9)-(14) into (8), after some simple transformations we arrive to the sensitivities sought

$$\frac{dz}{dh_1}=-\frac{h_1\left[zc\,tg(\alpha_1+\alpha_2+\alpha_3+\delta)+x\right]}{(a_1b_0-a_0b_1)sin\alpha_1+(a_0a_1+b_0b_1)\cos\alpha_1} \quad (15)$$

$$\frac{dz}{dh_2}=-\frac{(z-l_1 sin\alpha_1)c\,tg(\alpha_1+\alpha_2+\alpha_3+\delta)+x-l_1\cos\alpha_1}{a_2\cos(\gamma_0-\alpha_2)+c_2 sin(\gamma_0-\alpha_2)}\cdot\frac{h_2}{b_2} \quad (16)$$

$$\frac{dz}{dh_3}=\frac{\partial z}{\partial x}\frac{dx}{dh_3}+\frac{\partial z}{\partial\alpha_3}\left(\frac{\partial\alpha_3}{\partial h_3}+\frac{\partial\alpha_3}{\partial\beta}\frac{d\beta}{dh_3}\right) \quad (17)$$

$$\frac{dx}{dh_1}=\frac{zh_1}{(a_1b_0-a_0b_1)sin\alpha_1+(a_0a_1+b_0b_1)\cos\alpha_1} \quad (18)$$

$$\frac{dx}{dh_2}=\frac{l_2 sin(\alpha_1+\alpha_2)+l_3 sin(\alpha_1+\alpha_2+\alpha_3)}{c_2 sin(\gamma_0-\alpha_2)-a_2\cos(\gamma_0-\alpha_2)}\cdot\frac{h_2}{b_2} \quad (19)$$

$$\frac{dx}{dh_3}=-l_3 sin(\alpha_1+\alpha_2+\alpha_3+\delta)\cdot\frac{\partial\alpha_3}{\partial\beta}\cdot\frac{d\beta}{dh_3} \quad (20)$$

4. SENSITIVITIES FOR AN OPTIMUM TRAJECTORY

In this section numerical results for sensitivities of z and x belonging to an optimum trajectory are given. The problem of an optimum trajectory is stated as follows (for details see [3])

The bucket that moves in such a way that its tip is traveling along a trajectory which length has to be minimized. In an unconstrained motion, the minimum length trajectory is a straight line joining the two considered points. In a real excavation however, this may not be the case. This is because of several constraints imposed on the bucket motion. They are:

- During each cut the bucket has to be filled to a given volume V_o;
- The bucket motion is constrained by limited lengths of actuators. This in turn is limiting rotations α_1, α_2 of mechanism arms and α_3 of the bucket.
- The optimized digging trajectory z (x) depends on the shape of the free ground level $z''(x)$ surface.
- The bucket edge starts to penetrate the earth along the vertical line of the excavation boundary. The maximum available penetration depth d depends on the kind of the soil.
- The bucket edge should stay in the soil traveling between two points of the trajectory.

Transforming the above into the formal relations for a finite number of points at x axis, we arrive to the following nonlinear problem:

Minimize:

$$S=\sum_{i=1}^{i=i_0}\sqrt{(z_i-z_{i-1})^2+(x_i-x_{i-1})^2} \quad (21)$$

subjected to the constrains

$$V_0=\frac{1}{2}\sum_{i=1}^{i=i_0}(z_i-z_{i-1})\cdot(x_i-x_{i-1}) \quad (22)$$

$$x_i=l_1\cos(\alpha_1)+l_2\cos(\alpha_1+\alpha_2)+ l_3\cos(\alpha_1+\alpha_2+\alpha_3) \quad (23)$$

$$z_i=l_1 sin(\alpha_1)+l_2 sin(\alpha_1+\alpha_2)+ l_3 sin(\alpha_1+\alpha_2+\alpha_3) \quad (24)$$

$$\frac{z_i - z_{i-1}}{x_i - x_{i-1}} = -c\,\mathrm{tg}(\alpha_{1i} + \alpha_{2i} + \alpha_{3i} + \delta) \qquad (25)$$

$$\alpha_{1min} \leq \alpha_{1i} \leq \alpha_{1max}; \quad \alpha_{2min} \leq \alpha_{2i} \leq \alpha_{2max}; \quad \alpha_{3min} \leq \alpha_{3i} \leq \alpha_{3max}; \qquad (26)$$

$$0 \leq z_i \leq d_i \qquad (27)$$

An optimum trajectory for small backhoe excavator with the following parameters:

$a_0 = 0.120$ [m]; $a_1 = 0.300$ [m]; $a_2 = 0.658$ [m]; a3 = 0.269 [m];
$b_0 = 0.300$ [m]; $b_1 = 1.120$ [m]; $b_2 = 0.300$ [m]; $b_3 = 0.190$ [m];
$c_1 = 0.941$ [m]; $c_2 = 1.037$ [m]; $c_3 = 0.175$ [m];
d = 0.260 [m]; g = 0.2 [m]; f = 0.175 [m]; e = 0.199 [m];
l1 = 2.200 [m]; $l_2 = 1.100$ [m]; l3 = 0.700 [m];
$\gamma_0 = 0.0$ [rad]; $\mu = 1.676$ [rad]; $\delta = 0.0$ [rad]

is presented in Fig 5:

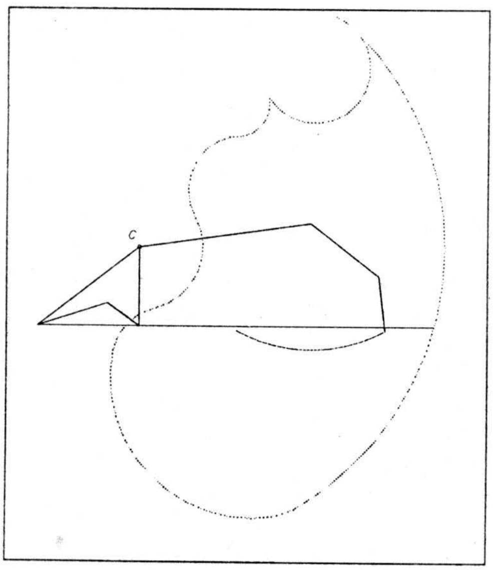

Fig. 5 The optimum trajectory

Substituting obtained values of z_i for particular x_i in equations (15) – (16), we find sensitivities of z with respect to h_1 and h_2.. They are presented in Fig.6.

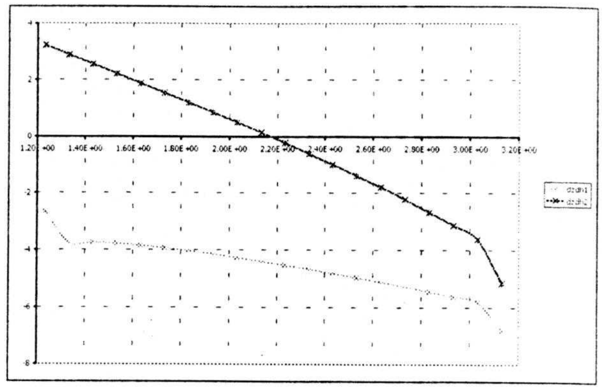

Fig. 6. Sensitivities dz/dh_1 dz/dh_2 for the optimum trajectory

5. SENSITIVITIES FOR A LINE *z=cosnt.*

Assume that the bucket tip is travelling along the line $z=z_0$. At the same time the bucket is oriented by the *angle* $\alpha_1+\alpha_2+\alpha_3 = 0$. It means that the line joining the tip with the bucket linkage is vertical.
For the same excavator, with the parameters given in previous section, the functions dz/dh1 and dz/dh2 are presented in Fig.7.

6. CONCLUSIONS.

The obtained numerical result show that discussed sensitivities should not be neglected in planning controlled excavation along an assumed trajectory. In both discussed cases, the variation of actuator length can cause significant large variation of the tip z displacement. If for instance, in case of an optimum trajectory, the bucket is at the curved and $\Delta h1=\Delta h2=0.010$ [m] the total z variation can reach up to $\Delta z = 0.120 - 0.130$ [m]. It is interesting to note that the largest sensitivities for z= cons, are larger than the largest sensitivities for an optimum trajectory. However, this is comparison of two cases only and general conclusion of this fact can not be generalized. Relatively, observed large sensitivity should encourage for farther investigation of the problem including dynamic effect caused by values opening and closing.

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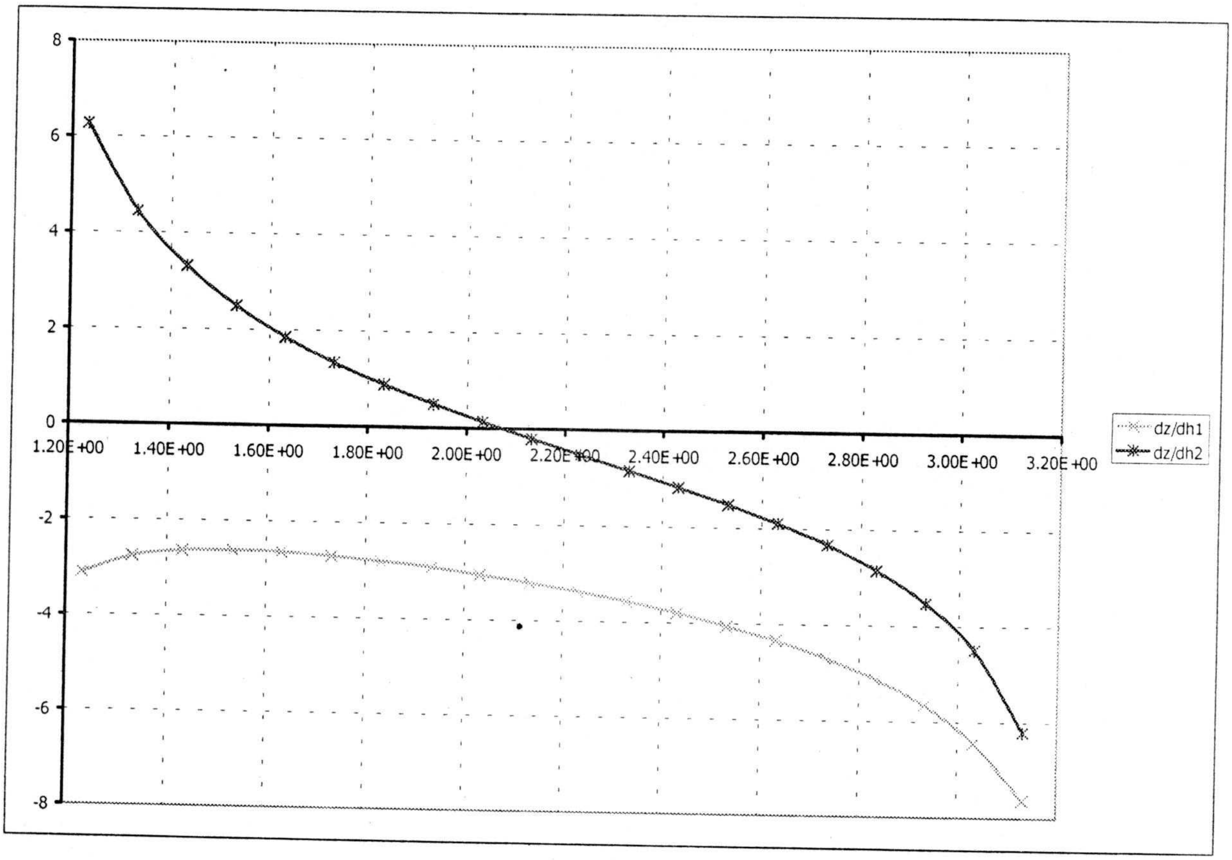

Fig. 7. Sensitivities fdz/dh1 and dz/dh2 for *z=const*.