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KINEMATIC FUNDAMENTALS FOR THE SEMI-AUTOMATED DRILLING OF VERTICAL BLAST HOLES IN SALT MINES

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To increase the accuracy of drilling in salt diapir a software module with kinematic equations was integrated into the control of a drilling vehicle. Despite the complex hybrid serial-parallel structure the HD-notation is used for the kinematic equations. DKP and IKP (with the method of Paul) are calculated analytically. The undefined ceiling of the tunnel requires a semi-automated operation which is enabled by a combination of DKP and IKP. Since reference points are unknown, a method was developed in which the reference points lie in a defined position to each other and to the reference system.

Keywords: Drilling vehicle, blast holes, salt mines, HD-notation, methods of Paul

1. INTRODUCTION

According to the drilling scheme shown below (Figure 1), blast holes P_i are drilled with the aid of a drilling vehicle to extract salt vertically. The height of the boreholes can be up to 100 m. To blast the salt in an optimal way angle α transverse the salt diapir and angle β along the salt diapir have to be kept within certain given limits. On account of the handicaps of the Mining Office, the width must not exceed 40 m.

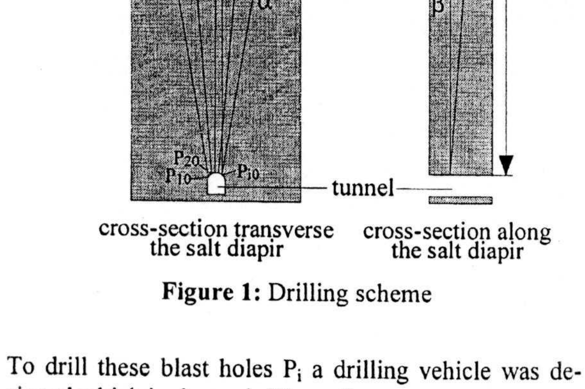

Figure 1: Drilling scheme

To drill these blast holes P_i a drilling vehicle was designed which is shown in Figure 2.

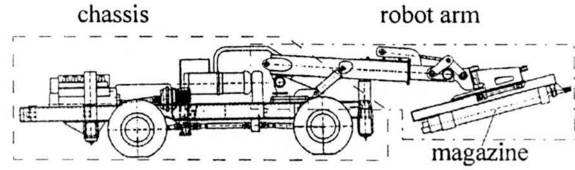

Figure 2: Design of the drilling vehicle

The drilling vehicle can be divided into two main modules: the chassis and the robot arm with the magazine holder. In this magazine the drilling bars are stored. These drilling bars have to be composed automatically by an operator until the desired drilling length l is reached.

To drill the boreholes according to the scheme described above the top of the magazine holder has to correspond to point P_{i0}, the orientation of the magazine holder has to correspond to the angles α und β. When the drilling bars are extended to the drilling length l the point P_i is reached.

In the case of a previous drilling vehicle design the robot arm and the magazine holder were adjusted by the operator. So the experience of the operator determines the accuracy. To increase the accuracy of the angles α and β and of the position P_i a new design of the drilling vehicle was developed. The robot arm and the magazine holder of this new vehicle now can be moved semi-automated. The semi-automated operation is possible because of a control that includes a software-modul based on the following considerations.

2. KINEMATICS OF THE DRILLING VEHICLE

2.1 Movement possibilities

The kinematic structure of the robot arm and of the magazine holder can be divided into serial and parallel subchains which lie in two different planes (Figure 3).

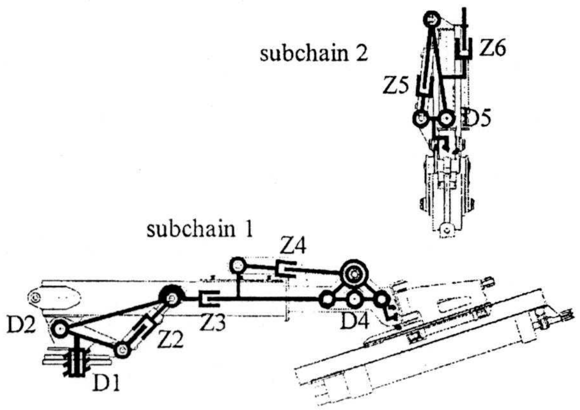

Figure 3: Kinematic structure

These subchains consist exclusively of rotary and prismatic joints and form planar mechanisms individually. The joints have one degree of freedom ($f_i = 1$):

$$F_{plane\,1} = F_{plane\,2} = 3 \cdot (n - g - 1) + \sum f_i - f_{id} \qquad [1]$$

with n: number of links g: number of joints
Σf_i: sum of the degrees of freedom
f_{id}: identical degrees of freedom

plane 1: $n = 9$	plane 2: $n = 5$
$g = 10$	$g = 5$
$\Sigma f_i = 10$	$\Sigma f_i = 5$
$f_{id} = 0$	$f_{id} = 0$
$F_{plane\,1} = 4$	$F_{plane\,2} = 2$

$F_{total} = F_{plane\,1} + F_{plane\,2} = 6$

Consequently, each point within the range of the drilling vehicle can be reached in any orientation of the magazine holder, e. g. robot arm and magazine holder can be moved as follows:

Movement	rotary joint	elevating ram
Swing robot arm	D1	-
Lift/lower robot arm	D2	Z2
Telescope robot arm	-	Z3
Lift/lower magazine holder	D4	Z4
Swing magazine holder	D5	Z5
Telescope magazine holder	-	Z6

The parallel subchains serve merely to transform the translation of the elevating rams Z2, Z4 and Z5 into a rotation of the rotary joint Di with corresponding force amplification. Since a rotationally symmetrical drilling tool is at the top of the magazine holder, a degree of freedom of $F = 5$ would be sufficient for drilling. For example the movement of the elevating ram Z6 is not necessary.

2.2 Calculation of the position and of the orientation of the magazine holder depending on the axis coordinates (Direct Kinematic Problem DKP)

Since the parallel subchains have no effect on the kinematics of the vehicle, they can be neglected. Therefore, the kinematics of the drilling vehicle can be represented as in Figure 4.

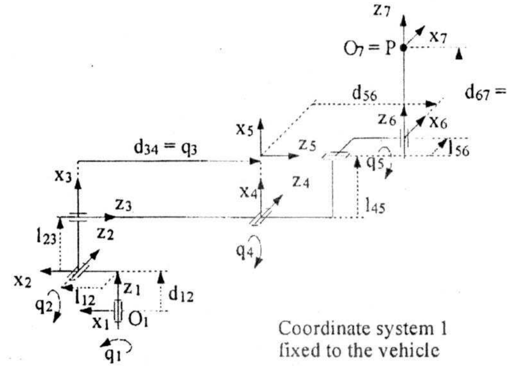

Figure 4: HD-notation of the drilling vehicle

The rotary and prismatic joints of the drilling vehicle are equipped with measuring systems. These systems measure the angles and positions of the axis coordinates $\underline{q}$. Using the measurement results the general position vector $\underline{\rho}$ (position of the point P at the top of the magazine holder and orientation of the magazine holder) can be calculated referring to the coordinate system 1 fixed to the vehicle:

$$\underline{\rho} = \left[x_P, y_P, z_P, \psi_P, \vartheta_P, \varphi_P\right]^T = f(\underline{q})$$

with $\underline{q} = \left[q_1, q_2, q_3, q_4, q_5, q_6\right]^T$

and $\psi_P, \vartheta_P, \varphi_P$

$\psi_P, \vartheta_P, \varphi_P$ are the Euler angles (rotation around the z-, x- and z-axis).

The HD-notation [2] is applied to get the solution:

$$^1\underline{\underline{H}}_P = {^1\underline{\underline{H}}_7} = \prod_{i=1}^{6} {^i\underline{\underline{H}}_{i+1}}(q_i) = \begin{bmatrix} ^1\underline{\underline{R}}_7 & ^1\underline{r}_{O,P} \\ \underline{0}^T & 1 \end{bmatrix}$$

$$= {^1\underline{\underline{H}}_2}(q_1) \cdot {^2\underline{\underline{H}}_3}(q_2) \cdot \ldots \cdot {^6\underline{\underline{H}}_7}(q_6)$$

with $^1\underline{\underline{R}}_7$: rotaton matrix
$^1\underline{r}_{O1P}$: translation vector

and $$ {}^{i}\underline{\underline{H}}_{j} = \begin{bmatrix} {}^{i}\underline{\underline{R}}_{j} & {}^{i}\underline{r}_{OiOj} \\ \underline{0}^{T} & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} \cos\delta_{ij} & -\sin\delta_{ij}\cdot\cos\lambda_{ij} & \sin\delta_{ij}\cdot\sin\lambda_{ij} & l_{ij}\cdot\cos\delta_{ij} \\ \sin\delta_{ij} & \cos\delta_{ij}\cdot\cos\lambda_{ij} & -\cos\delta_{ij}\cdot\sin\lambda_{ij} & l_{ij}\cdot\sin\delta_{ij} \\ 0 & \sin\lambda_{ij} & \cos\lambda_{ij} & d_{ij} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The HD-notation supplies the position ${}^{1}\underline{r}_{O1P}$ of the point P at the top of the magazine holder and orientation ${}^{1}\underline{\underline{R}}_{7}$ of the magazine holder.

2.3 Calculation of the axis coordinates depending on the position and orientation of the magazine holder (Inverse Kinematic Problem IKP)

If the drilling operation shall be carried out fully automatically, the position (${}^{1}\underline{r}_{O1P}$) and the orientation (${}^{1}\underline{\underline{R}}_{7}$) of the magazine holder can be pre-set. Since the distance for the 6th axis has not to be calculated (see 2.1), the measured value of this axis is constant in the calculation.

Given: $x_P, y_P, z_P, \psi_P, \vartheta_P, q_6$

Besides of the axis coordinates $q_1...q_5$ the angle of orientation φ_P has to be calculated:

Unknown: $q_1, q_2, q_3, q_4, q_5, \varphi_P$

Using equation

$$ {}^{1}\underline{\underline{H}}_{P} = \begin{bmatrix} \alpha' & \alpha'' & \alpha''' & x_P \\ \beta' & \beta'' & \beta''' & y_P \\ \gamma' & \gamma'' & \gamma''' & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{1}\underline{\underline{H}}_{2}(q_1)\cdot\ldots\cdot{}^{6}\underline{\underline{H}}_{7}(q_6) \quad (1) $$

$\alpha', \alpha'', \ldots, \gamma'', \gamma'''$ depend on the Euler angles ψ_P, ϑ_P (given) and the orientation angle φ_P (unknown). Eq. (1) can be written as

$$ {}^{1}\underline{\underline{H}}_{P} = \begin{bmatrix} m\cdot c\varphi_P + n\cdot s\varphi_P & -m\cdot s\varphi_P + n\cdot c\varphi_P & \alpha''' & x_P \\ p\cdot c\varphi_P + r\cdot s\varphi_P & -p\cdot s\varphi + r\cdot c\varphi_P & \beta''' & y_P \\ d\cdot s\varphi_P & d\cdot c\varphi_P & \gamma''' & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2) $$

with
$c\varphi_P = \cos\varphi_P$ $\quad s\varphi_P = \sin\varphi_P$
$m = \cos\psi_P$ $\quad n = -\sin\psi_P\cdot\cos\vartheta_P$
$p = \sin\psi_P$ $\quad r = \cos\psi_P\cdot\cos\vartheta_P$
$d = \sin\vartheta_P$
$\alpha''' = \sin\psi_P\cdot\sin\vartheta_P$ $\quad \beta''' = -\cos\psi_P\cdot\sin\vartheta_P$
$\gamma''' = \cos\vartheta_P$

Since the axis coordinate q_6 is given and coordinate system 6 has the same orientation as coordinate system 7 Eq. (1) with Eq. (2) can be simplified:

$$ {}^{1}\underline{\underline{H}}_{6} = \begin{bmatrix} m\cdot c\varphi_P + n\cdot s\varphi_P & -m\cdot s\varphi_P + n\cdot c\varphi_P & \alpha''' & x_{O6} \\ p\cdot c\varphi_P + r\cdot s\varphi_P & -p\cdot s\varphi + r\cdot c\varphi_P & \beta''' & y_{O6} \\ d\cdot s\varphi_P & d\cdot c\varphi_P & \gamma''' & z_{O6} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3) $$

$$ = {}^{1}\underline{\underline{H}}_{2}(q_1)\cdot{}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4)\cdot{}^{5}\underline{\underline{H}}_{6}(q_5) $$

with $x_{O6} = x_P - q_6\cdot\alpha'''$ $\quad y_{O6} = y_P - q_6\cdot\beta'''$
$z_{O6} = z_P - q_6\cdot\gamma'''$

The method of Paul [2] is used for the solution. Due to this method Eq. (3) is varied by multiplication with the inverse matrices:

$$ {}^{1}\underline{\underline{H}}_{6}(\varphi_P) = {}^{1}\underline{\underline{H}}_{2}(q_1)\cdot{}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4)\cdot{}^{5}\underline{\underline{H}}_{6}(q_5) $$
$$ {}^{1}\underline{\underline{H}}_{6}(\varphi_P)\cdot{}^{5}\underline{\underline{H}}_{6}^{-1}(q_5) = {}^{1}\underline{\underline{H}}_{2}(q_1)\cdot{}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4) $$
$$ {}^{1}\underline{\underline{H}}_{6}(\varphi_P)\cdot{}^{5}\underline{\underline{H}}_{6}^{-1}(q_5)\cdot{}^{4}\underline{\underline{H}}_{5}^{-1}(q_4) = {}^{1}\underline{\underline{H}}_{2}(q_1)\cdot{}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3) $$
etc.
$$ {}^{1}\underline{\underline{H}}_{2}^{-1}(q_1)\cdot{}^{1}\underline{\underline{H}}_{6}(\varphi_P) = {}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4)\cdot{}^{5}\underline{\underline{H}}_{6}(q_5) $$
$$ {}^{2}\underline{\underline{H}}_{3}^{-1}(q_2)\cdot{}^{1}\underline{\underline{H}}_{2}^{-1}(q_1)\cdot{}^{1}\underline{\underline{H}}_{6}(\varphi_P) = {}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4)\cdot{}^{5}\underline{\underline{H}}_{6}(q_5) $$
etc.
$$ {}^{1}\underline{\underline{H}}_{2}^{-1}(q_1)\cdot{}^{1}\underline{\underline{H}}_{6}(\varphi_P)\cdot{}^{5}\underline{\underline{H}}_{6}^{-1}(q_5) = {}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3)\cdot{}^{4}\underline{\underline{H}}_{5}(q_4) $$
$$ {}^{1}\underline{\underline{H}}_{2}^{-1}(q_1)\cdot{}^{1}\underline{\underline{H}}_{6}(\varphi_P)\cdot{}^{5}\underline{\underline{H}}_{6}^{-1}(q_5)\cdot{}^{4}\underline{\underline{H}}_{5}^{-1}(q_4) = {}^{2}\underline{\underline{H}}_{3}(q_2)\cdot{}^{3}\underline{\underline{H}}_{4}(q_3) $$
etc.

31 different equations result from these multiplications. 12 matrix elements can be compared on both sides of each equation. Those equations which enable an analytical solution are selected from these $12 \times 31 = 372$ equations.
In the case of the drilling vehicle this analytical solution leads to 16 positions which can be limited by steric conditions, e. g. by the pre-set range of the drilling vehicle.

3. OPERATING MODES

The operator can select between two operating modes: the manual operation and the semi-automated fixed-point operation.

3.1 Manual operation

With this operating mode the operator adjusts all axes of the drilling vehicle with remote control. From the current measured values for the axis coordinates $\underline{q}$ the control calculates the angles $\alpha_{current}$ and $\beta_{current}$ using the DKP (see 2.2). If a file with the coordinates of the points P_i of the drilling scheme is integrated into the control, the current values $\alpha_{current}$ and $\beta_{current}$ can be compared directly with the the reference values $\alpha_{reference}$ and $\beta_{reference}$. If it is necessary the operator can correct the axes.

3.2 Semi-automated fixed-point

Using the IKP (see 2.3) the axis coordinates $\underline{q}$ can be moved fully automatically, if the points P_i or P_{i0} and the angles α and β are given. For safety reasons the drilling vehicle must not be operated full-automated, since the steric conditions of the tunnel limit the operation. Therefore, a semi-automated so-called fixed-point operation was integrated instead of a full-automated operation.

The fixed-point operation requires a file with the points P_i of a drilling scheme. This file has to be integrated into the control.
The operator sets up the four axes with remote control:

- Swing robot arm (q_1)
- Lift/lower robot arm (q_2)
- Telescope robot arm (q_3)
- Telescope magazine holder (q_6)

From the current measured values of the four axes and from the given position of point P_i the control computes the necessary angles for

- lift/lower magazine holder (q_4) and
- swing magazine holder (q_5).

The control adjusts automatically these two angles. The current values $\alpha_{current}$ and $\beta_{current}$ can be compared directly with the the reference values $\alpha_{reference}$ and $\beta_{reference}$. If it is necessary the operator can correct the four axes.

Figure 5 clarifies the computations practised in the control:

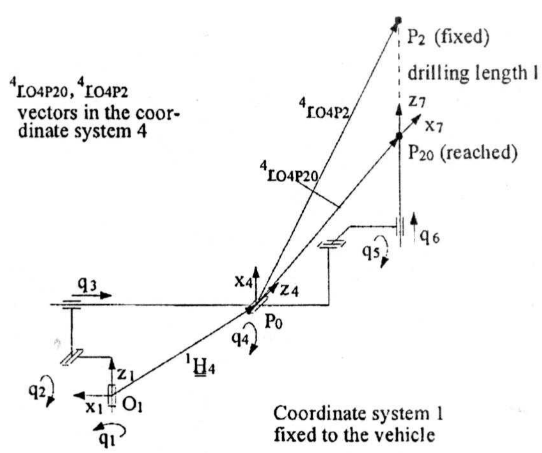

Figure 5: Fixed-point operation

The position ${}^1\underline{r}_{O1O4}$ and the orientation ${}^1\underline{\underline{R}}_{O1O4}$ of the origin O_4 can be calculated with the given angles and positions q_1, q_2 and q_3:

$$ {}^1\underline{\underline{H}}_4 = f\left(q_1, q_2, q_3\right) \qquad \text{(see DKP, 2.2)} $$

The coordinates of the fixed point P_2 are given in coordinate system 1. These coordinates can be transformed into coordinate system 4:

$$ {}^1\underline{r}_{P2} = {}^1\underline{r}_{O1O4} + {}^1\underline{r}_{O4P2} = {}^1\underline{r}_{O1O4} + {}^1\underline{\underline{H}}_4 \cdot {}^4\underline{r}_{O4P2} \tag{4} $$

The direction of the magazine holder corresponds to the orientation of the ${}^7z_7$-axis in coordinate system 7 at the top of the magazine holder.
In coordinate system 4 the following Eq. (5) results for the position ${}^4\underline{r}_{O4P2}$ and the orientation ${}^4\underline{\underline{R}}_{O4P20}$ of point P_{20} at the top of the magazine holder:

$$ {}^4\underline{\underline{H}}_7 = {}^4\underline{\underline{H}}_5 \cdot {}^5\underline{\underline{H}}_6 \cdot {}^6\underline{\underline{H}}_7 = \begin{array}{c} \begin{bmatrix} \bullet & \bullet & \alpha''' & {}^4x_{P20} \\ \bullet & \bullet & \beta''' & {}^4y_{P20} \\ \bullet & \bullet & \gamma''' & {}^4z_{P20} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \qquad\qquad \uparrow \qquad \uparrow \\ \text{direction of } {}^4z_7 \quad {}^4\underline{r}_{O4P20} \end{array} \tag{5} $$

with $\quad \alpha''', \beta''', \gamma''' = f(q_4, q_5)$
and $\quad {}^4\underline{r}_{O4P20} = f(q_4, q_5, q_6)$

With this we get

$$ {}^4\underline{r}_{O4P2} = {}^4\underline{r}_{O4P20} + {}^4\underline{r}_{P20P2} = $$
$$ \begin{bmatrix} {}^4x_{P20} \\ {}^4y_{P20} \\ {}^4z_{P20} \end{bmatrix} + \lambda \cdot \begin{bmatrix} \alpha''' \\ \beta''' \\ \gamma''' \end{bmatrix} = {}^1\underline{\underline{H}}_4^{-1} \cdot {}^1\underline{r}_{O4P2} \tag{6} $$

Eq. (5) and Eq. (6) lead to the solution for q_4, q_5 and λ. λ is simultanously the drilling length l (distance between the top of the magazine holder P_{20} and point P_2).

4. REFERENCE POINTS

The coordinates of the points P_i which have to be drilled are stored in a file. These coordinates refer to coordinate system 0 of the drilling scheme. However, the position and orientation of the drilling vehicle in coordinate system 0 is unknown. Therefore, the position and orientation of the vehicle (${}^0\underline{\underline{H}}_1$) must be calculated in coordinate system 0 of the drilling scheme (Figure 6).

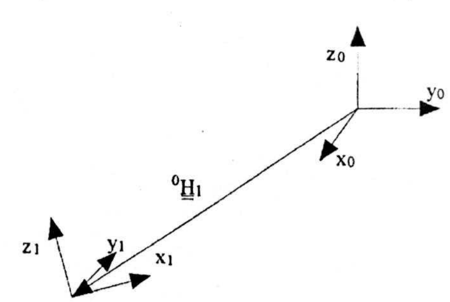

Figure 6: Vehicle-fixed coordinate system 1 and coordinate system 0 of the drilling scheme

For the calculation of ${}^0\underline{\underline{H}}_1$ reference points can be used. The positions of these reference points have to be known in the drilling scheme 0 (${}^0\underline{r}_{\text{reference point}}$). Furthermore their positions in coordinate system 1 (${}^1\underline{r}_{\text{reference point}}$) can be calculated with the DKP if the axis coordinates $\underline{q}$ are measured. Several solutions are possible, e. g.:

- 4 points A, B, C and D are given

$$
{}^0\underline{\underline{H}}_1 = \begin{bmatrix} {}^0\underline{\underline{R}}_1 & {}^0\underline{r}_{O_0O_1} \\ \underline{0}^T & 1 \end{bmatrix}
= \begin{bmatrix} {}^0\underline{r}_A & {}^0\underline{r}_C & {}^0\underline{r}_B & {}^0\underline{r}_D \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} {}^1\underline{r}_A & {}^1\underline{r}_C & {}^1\underline{r}_B & {}^1\underline{r}_D \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}
$$

- 3 points A, B, C are given, point D is give by point C (reflected at $\overline{AB}$)

$$
{}^0\underline{\underline{R}}_1 = \left[{}^0\underline{r}_{AB} \quad {}^0\underline{r}_{AC} \quad {}^0\underline{r}_{AD} \right] \cdot \left[{}^1\underline{r}_{AB} \quad {}^1\underline{r}_{AC} \quad {}^1\underline{r}_{AD} \right]^{-1}
$$

$$
{}^0\underline{r}_{O_0O_1} = {}^0\underline{r}_A - {}^0\underline{\underline{R}}_1 \cdot {}^1\underline{r}_A
$$

The disadvantage of these solutions is the fact that the positions of the reference points must be known in coordinate system 0 of the drilling scheme. However, because of the different steric conditions of the tunnel it is not easy to determine such reference points (e.g. at the ceiling of the tunnel) and to move the robot arm in that way that the magazine holder reaches this reference point. Therefore following considerations lead to a solution.

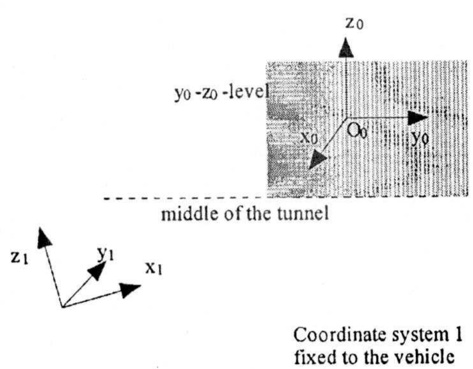

Figure 7: Origin of coordinate system 0 and the middle of the tunnel

The origin of coordinate system 0 is marked at the ceiling of the tunnel (Figure 7). Also the so-called middle of the tunnel is marked. It is located in the y_0-z_0-level of the drilling scheme, horizontal and therefore parallel to the y_0-axis.

We take from that the following (Figure 8):

- The three points A, B and C have to be reached with the magazine holder in vertical orientation. which is enabled by measuring systems and an adequate compensation control.
- The points A and B are located in the middle of the tunnel.
- The point C is the origin of coordinate system 0 (${}^0\underline{r}_C = \underline{0}$). It has to be reached directly after reaching point B by telescoping the magazine holder.

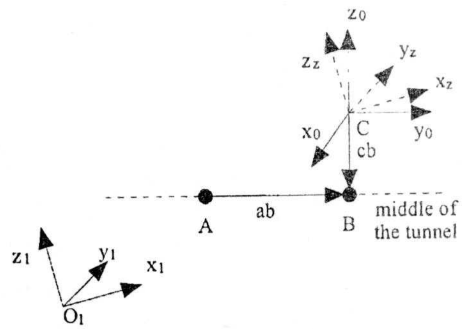

Figure 8: Reference points

If the points have been reached in the described way, the measuring systems supply the following values:

$$
{}^1\underline{r}_A = f(\underline{q}) \qquad {}^1\underline{r}_B = f(\underline{q}) \qquad {}^1\underline{r}_C = f(\underline{q})
$$

The intermediate coordinate systeme z has the same orientation as the vehicle-fixed coordinate systeme 1, but it has an offset of the vector ${}^1\underline{r}_C$. Therefore, it is clear that

$$
{}^z\underline{cb} = {}^z\underline{r}_B - {}^z\underline{r}_C = {}^1\underline{cb} = {}^1\underline{r}_B - {}^1\underline{r}_C = \begin{bmatrix} {}^z cb_x \\ {}^z cb_y \\ {}^z cb_z \end{bmatrix}
$$

$$
{}^z\underline{ab} = {}^z\underline{r}_B - {}^z\underline{r}_A = {}^1\underline{ab} = {}^1\underline{r}_B - {}^1\underline{r}_A = \begin{bmatrix} {}^z ab_x \\ {}^z ab_y \\ {}^z ab_z \end{bmatrix}
$$

${}^z cb_x$, ${}^z cb_y$, ${}^z cb_z$, ${}^z ab_x$, ${}^z ab_y$, ${}^z ab_z$ are given by the measuring systems of the axis coordinates $\underline{q}$.

In coordinate system 0 of the drilling scheme we get

$$
{}^0A = \begin{bmatrix} 0 \\ -{}^0ab \\ {}^0cb \end{bmatrix}, \qquad {}^0B = \begin{bmatrix} 0 \\ 0 \\ {}^0cb \end{bmatrix}, \qquad {}^0C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
$$

$\underline{cb}$ and $\underline{ab}$ are given by

$$
{}^0\underline{cb} = \begin{bmatrix} 0 \\ 0 \\ {}^0cb \end{bmatrix}, \qquad {}^0\underline{ab} = \begin{bmatrix} 0 \\ {}^0ab \\ 0 \end{bmatrix}.
$$

${}^0cb$ is given by telescoping the magazine holder. It is the difference between q_6 when reaching point B and q_6 when reaching point C. ${}^0ab$ is unknown.

The ${}^0x_1$-, ${}^0y_1$- and ${}^0z_1$-axis in the drilling scheme (coordinate system 0) result from the matrix ${}^0\underline{\underline{R}}_z$:

$$
{}^0\underline{\underline{R}}_z = {}^0\underline{\underline{R}}_1 = \begin{bmatrix} \alpha' & \alpha'' & \alpha''' \\ \beta' & \beta'' & \beta''' \\ \gamma' & \gamma'' & \gamma''' \end{bmatrix}
$$

$$
{}^0x_1 - \text{axis} \uparrow \quad \uparrow \quad \uparrow {}^0z_1 - \text{axis}
$$

$$
{}^0y_1 - \text{axis}
$$

With the scalar product the angle between two vectors (with regard to different coordinate systems) can be compared:

$$
{}^0\underline{cb} \cdot {}^0\underline{x}_1 = \begin{bmatrix} 0 \\ 0 \\ {}^0cb \end{bmatrix} \cdot \begin{bmatrix} \alpha' \\ \beta' \\ \gamma' \end{bmatrix} = {}^z\underline{bc} \cdot {}^1\underline{x}_1 = \begin{bmatrix} {}^zcb_x \\ {}^zcb_y \\ {}^zcb_z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^0\underline{cb} \cdot {}^0\underline{y}_1 = \begin{bmatrix} 0 \\ 0 \\ {}^0cb \end{bmatrix} \cdot \begin{bmatrix} \alpha'' \\ \beta'' \\ \gamma'' \end{bmatrix} = {}^z\underline{bc} \cdot {}^1\underline{y}_1 = \begin{bmatrix} {}^zcb_x \\ {}^zcb_y \\ {}^zcb_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
$$

$$
{}^0\underline{cb} \cdot {}^0\underline{z}_1 = \begin{bmatrix} 0 \\ 0 \\ {}^0cb \end{bmatrix} \cdot \begin{bmatrix} \alpha''' \\ \beta''' \\ \gamma''' \end{bmatrix} = {}^z\underline{cb} \cdot {}^1\underline{z}_1 = \begin{bmatrix} {}^zcb_x \\ {}^zcb_y \\ {}^zcb_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

These three equations lead to the solutions for γ', γ'' and γ'''.

The solutions for α', α'' ... β''' are given by

$$
{}^0\underline{bc} = {}^0\underline{\underline{R}}_z \cdot {}^z\underline{bc}\,, \qquad {}^0\underline{ab} = {}^0\underline{\underline{R}}_z \cdot {}^z\underline{ab}
$$

and

$$
\alpha'^2 + \alpha''^2 + \alpha'''^2 = 1, \qquad \alpha'^2 + \beta'^2 + y'^2 = 1
$$

$$
\alpha''^2 + \beta''^2 + y''^2 = 1, \qquad \alpha'''^2 + \beta'''^2 + y'''^2 = 1
$$

There are 16 solutions for the rotation matrix ${}^0\underline{\underline{R}}_z$.

Based on the following conditions these solutions can be minimized to one solution.

- The three axes form a right-hand system, i.e. all three axes are perpendicular to each other and correspond to the x-, y- and z-direction.
- If point A is closer to the drilling vehicle than point B then ${}^0ab > 0$ else ${}^0ab < 0$.
- The rotation matrix can also be determined by using Kardan angles (rotation of the z-, y- and x-axis). Due to the steric conditions of the tunnel the absolute value of the three angles is less than 45° (abs(γ, β, α) < 45).

The translation vector runs

$$
{}^0\underline{r}_{O_0O_1} = - {}^0\underline{\underline{R}}_1 \cdot {}^1\underline{r}_C
$$

Summary

This paper deals the kinematic equations of a drilling vehicle in the winning of salt. A software module based on these eqations is integrated into the control to increase the accuracy of drilling in the manual operation as well as in the semi-automated fixed-point operation.

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