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ANALYSIS OF TOLERANCES IN ROBOTIC MAPPING OF BUILDINGS

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Abstract

Autonomous Map-Making is becoming a widely used tool in robotics for various applications. One of the major problems to be dealt with in the development of this tool is the tolerance of the coordinates obtained in the process of mapping. This paper presents an analysis of the tolerances of a map created by a robot of 6 articulated Degrees Of Freedom mounted on a mobile carriage and utilizing a laser beam range-finder for horizontal and vertical rotational scanning. The analysis shows that two parameters are the main factors affecting the tolerances of the map: (1) Orientation of the carriage on which the robot is mounted, and (2) Distance between the sensor and the walls being scanned. Three main conditions need to be satisfied in order to achieve coordinate tolerances not worse than 3 cm are: 1. Carriage Orientation accuracy of at least 0.1°; 2. Carriage Location accuracy of at least 1 cm; and 3. Distance of not more than 3 m between the sensor and the walls being scanned.

1. Introduction

TAMIR - Technion Autonomous Multipurpose Interior Robot - is a robotic system developed at Technion - Israel Institute of Technology, for the execution of finishing tasks in building. Two unique features of a construction robot are its capability to map its working environment and plan the performance of its own tasks autonomously. A major aspect of these capabilities is the compatibility of tolerances of the coordinates obtained from the mapping process for the performance of its tasks. The tolerances of the objects included in the map determine the suitability of the map for <u>autonomous navigation</u>, for <u>planning of the robotic task</u>, and for the <u>performance and monitoring of the robot task</u>.

The compatibility of technological tolerances [3, 4] was a major issue that had to be considered in the development of the mapping system. This paper presents an analysis of tolerances in mapping by a laser range finder attached as an end-effector to a robot of 6 articulated DOF mounted on a mobile carriage. The effects of the different parameters on the final resolution of the map of the enclosure is analyzed, the two main variables used as criteria in the evaluation of tolerances being: (1) Coordinates of corners of rooms; and (2) Coordinates of edges of openings. The resolutions of these variables determine the adequacy of the map obtained for the tasks mentioned. This paper focuses on the tolerance of the enclosure, while the entire analysis will be presented in a different publication.

2. Effect of Mapping Tolerances on the Planning of the Robotic Task

The effects of the tolerances of the mapping process on the suitability of the map for task planning depend on the type of task to be executed by the robot. Two main categories of tasks were defined [1]:

(1) Tasks that do not demand contact between the end-effector and the object dealt with (e.g. wall or ceiling painting). The tasks can be accomplished in coordinates given by a map with a resolution of up to 3 cm.

(2) Tasks that require direct contact between the end-effector and the object dealt with (e.g. building of wall partitions, or tile setting). This category of tasks demands a high resolution of both the map of the environment and the position of the end-effector (of the order of 2-3 mm).

The latter constraint is reasonable having regard to the precision required of the robot's end-effector, but it seems beyond the current state of the art in map-making. The analysis was therefore directed at the constraint set by the first category of tasks, the demands of a resolution of 3 cm with respect to the coordinates of both the building enclosure and its openings. The conceptual assumption at this stage is that the second category of tasks can be performed according to the same map with the addition of frequent calibrations during the task performance, done either automatically or semi-automatically.

3. Definition of the Building's Interior Environment

Several common features of building interiors to be processed by the interior finishing robot were defined in order to reduce the complexity of the mapping process [1]:

1. Building spaces are enclosed by vertical walls.
2. The walls are orthogonal with respect to each other, they are planar, and their surfaces are clear of exposed studs.
3. Edges of openings are orthogonal, and parallel to the edges of walls.
4. Ceilings and floors are horizontal, planar, and at a common elevation.
5. All rooms are closed orthogonal polygons.
6. The exterior enclosure of the building has already been completed, prior to the mapping.

4. Tolerances of Sensor Readings

Figure 1(a) schematically illustrates sensor readings from the end of the robot arm, while the robot itself is mounted on a mobile carriage. The robot is located at point (Xo, Yo) in the global coordinate system, the position of the Tool Center Point, at the end of the robot arm being assumed to be identical with the carriage location. The coordinates of the measured point (X_p, Y_p) at the global system are therefore (see Legend of Fig. 1):

$$X_p = X_o + R\cos(\theta_o + \alpha) \quad (1)$$

$$Y_p = Y_o + R\sin(\theta_o + \alpha) \quad (2)$$

Each of the 5 variables (X_o, Y_o, R, θ_o, α) has its own tolerance. For small dimensional variations, the Taylor's series expansion of equations (1) and (2) - neglecting the second and higher-order terms - yields equations (3) and (4) ([3],[4],[5]).

$$(\sigma X_p)^2 = (\frac{\partial X_p}{\partial X_o})^2 * (\sigma_{X_o})^2 + (\frac{\partial X_p}{\partial R})^2 * (\sigma R)^2 + (\frac{\partial X_p}{\partial \theta_o})^2 * (\sigma\theta_o)^2 + \frac{\partial X_p}{\partial \alpha} * (\sigma\alpha)^2 \quad (3)$$

$$(\sigma Y_p)^2 = (\frac{\partial Y_p}{\partial Y_o})^2 * (\sigma Y_o)^2 + (\frac{\partial Y_p}{\partial R})^2 * (\sigma R)^2 + (\frac{\partial Y_p}{\partial \theta_o})^2 * (\sigma\theta_o)^2 + (\frac{\partial Y_p}{\partial \alpha})^2 * (\sigma\alpha)^2 \quad (4)$$

where σX_p and σY_p are the evaluated tolerances of X_p and Y_p, and σX_o, σY_o, σR, $\sigma\theta_o$ and $\sigma\alpha$ are the average tolerance of each variable.

The values of these variables are as described hereunder:

1. **The tolerance of the carriage location is assumed to be 1 cm in each direction, $\sigma X_o = \sigma Y_o = 1$ cm (this assumption may lead to a maximum total location tolerance of the carriage of 1.4 cm = $\sqrt{\sigma Xo^2 + \sigma Yo^2} = \sqrt{1^2 + 1^2}$.**

2. **The radial tolerance of the laser range finder (based on multiple test readings towards the same point) $\sigma R = 1.2$ cm.**

3. **The angular resolution of the laser range finder was calculated as a constant, $\sigma\alpha = 0.1°$ (its actual orientation versus its assumed orientation).**

For the purpose of demonstration let us assume a point P with the following data:

Xo=300 cm; Yo=300 cm; $\theta o = 35°$; $\sigma\theta_o = 0.3°$; $\alpha = 10°$; R=500 cm

The tolerance of the point P is:

$$\sigma P = \sqrt{\sigma Xp^2 + \sigma Yp^2} = 3.66 \text{ cm} \quad (5)$$

The main contributors to that tolerance are:

1. **Location and orientation of the carriage.**
2. **The radial distance of the reading.**

5. Tolerance of the Entire Map

The hierarchic mapping carried out by the mapping system of TAMIR creates a data base in which a room is represented two-dimensionally by the coordinates of its corners, and the openings are represented by the layout of their location [5]. The tolerances of the mapping of the building's enclosure are for the most part determined by these two variables. The sources of the tolerances in the mapping of corners and edges of openings are the following:

1. Radial resolution of the sensor readings ($\sigma R=1.2$).
2. Angular tolerance of the sensor readings ($\sigma\alpha=0.1°$).
3. Tolerance of the carriage location ($\sigma Xo=\sigma Yo=1.0$ cm).
4. Tolerance of the carriage orientation.
5. Tolerance of the wall orientation.
6. Tolerance of the wall location (Translation).

The last three items will be discussed in the following paragraphs.

Figure 1(b) schematically describes the transformation of the axis of a wall being scanned from a local scanning station S_L, situated in the global coordinate system S_G. For low values of θ_o (which is always the case while the carriage moves parallel to the walls of the rooms), the equations expressing the walls' axes are:

The horizontal wall: $Y=Yo+a_1 \cos \theta_o + (b_1+tg\ \theta_o) X$ (6)

The vertical wall: $X=Xo+a_2 \cos \theta_o + (b_2-tg\ \theta_o) Y$ (7)

where:

Xo and Yo are the coordinates of the scanning station.

a_1, b_1, a_2 and b_2 are coefficients of the walls' equations in the global system.

θ_o = is the orientation of the scanning station in the global system.

The coordinates of the corner (Xc and Yc) are derived from the intersection of these two axes, and equations (6) and (7) yield:

$$Xc = Xo + a_2 \cos\theta_o + (b_2 - tg\theta_o)[Yo + a_1 \cos\theta_o + (b_1 + tg\theta_o)Xc] \qquad (8)$$

$$Yc = Yo + a_1 \cos\theta_o + (b_1 + tg\,\theta_o) * \{Xo + a_2 \cos\theta_o + (b_2 - tg\,\theta_o) * [Yo + a_1 \cos\theta_o + (b_1 + tg\,\theta_o)Xc]\} \qquad (9)$$

whence the coordinates of the corner in the global system can be obtained. The tolerance of the corner location is derived from the errors of the parameters in the equations of the wall axes (a_1, a_2, b_1, b_2) and from the errors of the location and orientation of the carriage. Taylor's expansion series, omitting the second and higher order terms, yields expressions (10) and (11):

$$(\sigma Xc)^2 = (\frac{\partial Xc}{\partial a_1})^2 * (\sigma a_1)^2 + (\frac{\partial Xc}{\partial a_2})^2 * (\sigma a_2)^2 + (\frac{\partial Xc}{\partial b_1})^2 * (\sigma b_1)^2 + (\frac{\partial Xc}{\partial b_2})^2 * (\sigma b_2)^2 +$$

$$(\frac{\partial Xc}{\partial \theta_o})^2 * (\sigma\theta_o)^2 + (\frac{\partial Xc}{\partial Xo})^2 * (\sigma Xo)^2 + (\frac{\partial Xc}{\partial Yo})^2 * (\sigma Yo)^2 \quad (10)$$

$$(\sigma Yc)^2 = (\frac{\partial Yc}{\partial a_2})^2 * (\sigma a_2)^2 + (\frac{\partial Yc}{\partial a_1})^2 * (\sigma a_1)^2 + (\frac{\partial Yc}{\partial b_1})^2 * (\sigma b_1)^2 + (\frac{\partial Yc}{\partial b_2})^2 * (\sigma b_2)^2 +$$

$$(\frac{\partial Yc}{\partial \theta_o})^2 * (\sigma\theta_o)^2 + (\frac{\partial Yc}{\partial Xo})^2 * (\sigma Xo)^2 + (\frac{\partial Yc}{\partial Yo})^2 * (\sigma Yo)^2 \quad (11)$$

σa_1, σa_2, σb_1 and σb_2 are the standard deviations of the parameters of the wall axes based on laboratory experiments with the laser range finder. The experiments consisted of repeated measurements carried out in front of a wall. These measured points on the 2D coordinate system are spread along the wall axis. The axis line is determined by a linear regression of the measured points, which produces an evaluation of the parameters of the line as well as the variances of these parameters. The errors in the rest of the variables - Xo, Yo, and θ_o - were determined experimentally.

Figure 2 contains 4 different graphs, which describe the dependence of the corner location tolerance on that of the carriage orientation ($\sigma\theta_o$), the carriage location tolerance (σXo, σYo) being assumed to be 1 cm. Each of these graphs relates to a given distance between the wall and the carriage (3, 4, 5 and 10 m respectively). The minimum tolerance in the coordinates of the corner is seen to be 2.8 cm (even when $\sigma\theta_o$ equals zero). Its sources are the tolerances of the readings taken by the sensor in conjunction with the measuring system (the robot arm). When the tolerance of the carriage orientation attains 0.2° and the distance of the robot does not exceed 3 m, the corner location tolerance is found to be about 3 cm. In addition it is seen that the distance between the wall and the carriage strongly affect the tolerance of the corner; increasing the distance between the carriage and the wall to 5 m raises the minimum corner tolerance to 4 cm, and at distance of 10 m the corner tolerance exceeds 7 cm. It is shown that the combination of carriage orientation and wall distance has a strong effect on the tolerance of the corner. The slope of the graph steepens as the distance from the wall increases.

Figure 3 shows the corner-tolerance dependence on the carriage-location tolerance (σXo, σYo), while the carriage orientation tolerance ($\sigma\theta_o$) is assumed to be 0.1°. The distance of the carriage from the wall is as indicated for each of the graphs. These graphs show that both the carriage-location tolerance and the distance from the wall have an almost linear effect on the corner-location tolerance.

The summary of these groups of graphs is that the minimal requirements for mapping by the rotational scanning method with a corner-location accuracy of about 3 cm are:

1. Carriage location precision of 1 cm.
2. Carriage orientation precision of 0.1°.
3. The distance of the mapped wall from the sensor does not exceed 3 m.

6. Conclusion

Previous stages of development showed two orders of map resolution constraints: (1) 3 cm for tasks that do not demand contact between the end-effector and the object dealt with; and (2) 3 mm resolution for tasks that do demand such contact.

An analysis of practicability revealed that the 3 mm resolution for the second group of tasks is not attainable in the current state of the art, while the 3 cm resolution for the first group of tasks can be achieved by three technologically feasible requirements:

(1) A tolerance of not more than 0.1° in the carriage orientation.
(2) A tolerance of not more than 1 cm in the carriage location.
(3) A distance of not more than 3 m between the scanner and the wall being scanned.

A trade-off relationship exists between these three variables, thus they may compensate for each other.

References

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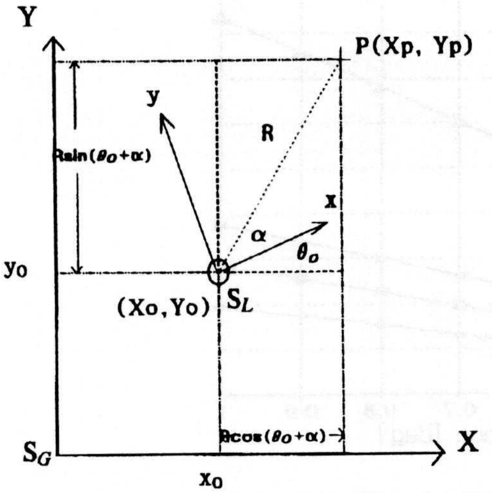

Figure 1(a): Interrelation between the sensor readings and the global coordinate system.

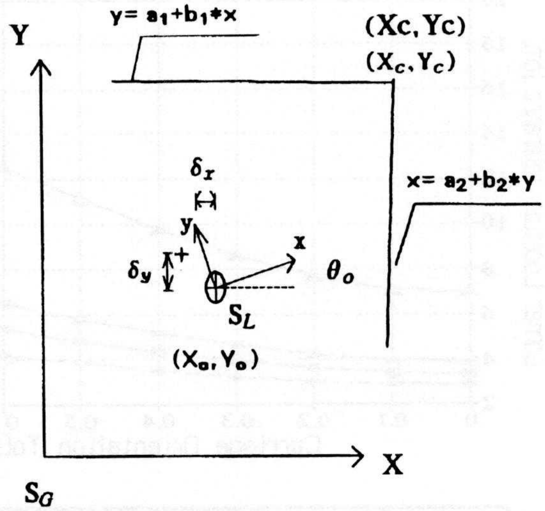

Figure 1(b): Calculation of room corner location by the intersection of two near-perpendicular walls

Legend:

X, Y – Global coordinate system.

x, y – Local coordinate system.

S_G – Origin of the global coordinate system.

S_L – Origin of the local coordinate system.

θ_o – Orientation of the robot in a scanning station in the global coordinate system.

x_o, y_o – Coordinates of robot location in the global coordinate system.

X_c, Y_c – Coordinates of corners in the local coordinate system.

Xc, Yc – Coordinates of the corners in the global coordinate system.

a_1, a_2, b_1, b_2 – Coefficients of wall equations in the local coordinate system.

δ_x, δ_y – Tolerance of robot location.

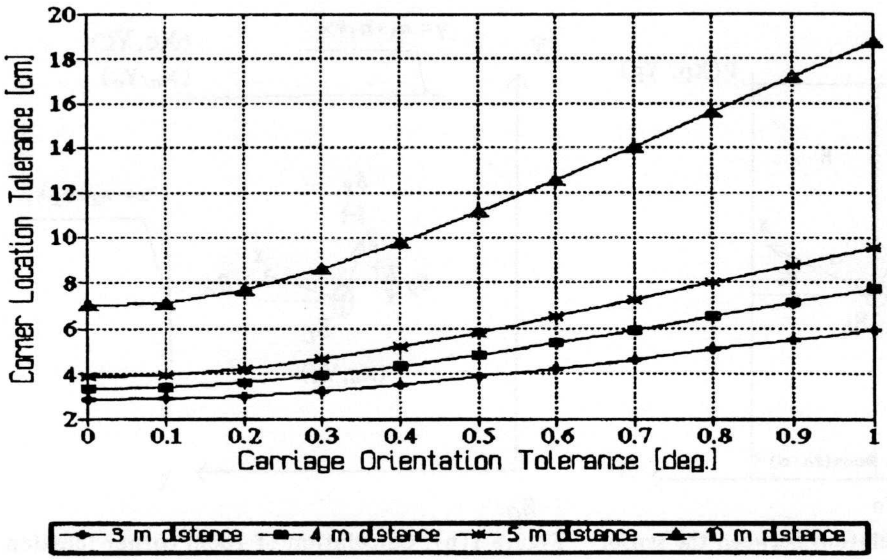

Fig 2: Room corner location tolerance as a function of carriage orientation tolerance, for 1 cm tolerance of carriage location and various values of wall distance.

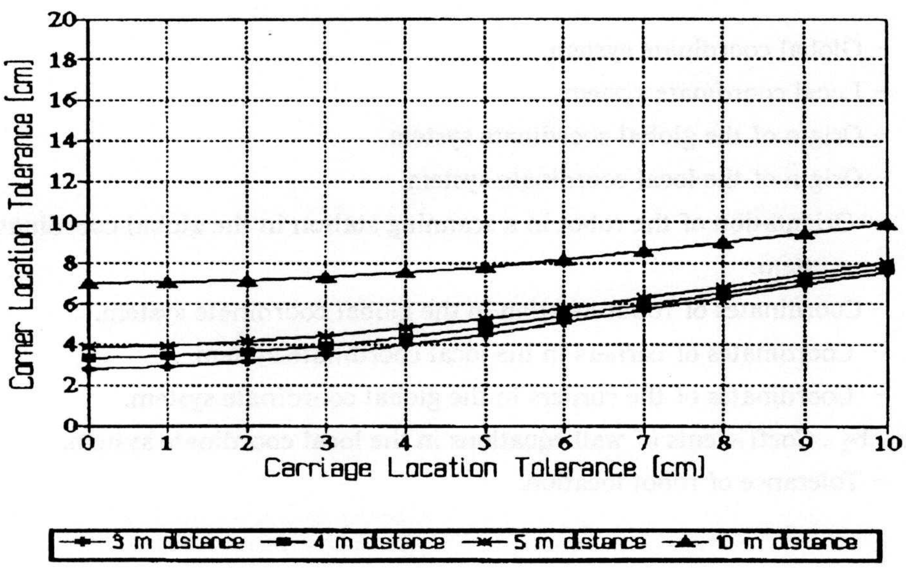

Fig. 3: Room-corner location tolerance as a function of carriage location tolerance, for 0.1^o tolerance of carriage orientation and various values of wall distance.